

<p>1(i) $f'(x) = \frac{(x^2+1)4x - (2x^2-1)2x}{(x^2+1)^2}$</p> $= \frac{4x^3 + 4x - 4x^3 + 2x}{(x^2+1)^2} = \frac{6x}{(x^2+1)^2} *$ <p>When $x > 0$, $6x > 0$ and $(x^2 + 1)^2 > 0$ $\Rightarrow f'(x) > 0$</p>	M1 A1 E1 M1 E1 [5]	Quotient or product rule correct expression www attempt to show or solve $f'(x) > 0$ numerator > 0 and denominator > 0 or, if solving, $6x > 0 \Rightarrow x > 0$
<p>(ii) $f(2) = \frac{8-1}{4+1} = 1\frac{2}{5}$</p> <p>Range is $-1 \leq y \leq 1\frac{2}{5}$</p>	B1 B1 [2]	must be \leq , y or $f(x)$
<p>(iii) $f'(x)$ max when $f''(x) = 0$</p> $\Rightarrow 6 - 18x^2 = 0$ $\Rightarrow x^2 = 1/3, x = 1/\sqrt{3}$ $\Rightarrow f'(x) = \frac{6/\sqrt{3}}{(1-\frac{1}{3})^2} = \frac{6}{\sqrt{3}} \cdot \frac{9}{16} = \frac{9\sqrt{3}}{8} = 1.95$	M1 A1 M1 A1 [4]	$(\pm)1/\sqrt{3}$ oe (0.577 or better) substituting $1/\sqrt{3}$ into $f'(x)$ $9\sqrt{3}/8$ o.e. or 1.95 or better (1.948557..)
<p>(iv) Domain is $-1 < x < 1\frac{2}{5}$</p> <p>Range is $0 \leq y \leq 2$</p>	B1 B1 M1 A1 cao [4]	ft their 1.4 but not $x \geq -1$ or $0 \leq g(x) \leq 2$ (not f) Reasonable reflection in $y = x$ from $(-1, 0)$ to $(1.4, 2)$, through $(0, \sqrt{2}/2)$ allow omission of one of $-1, 1.4, 2, \sqrt{2}/2$
<p>(v) $y = \frac{2x^2-1}{x^2+1}$ $x \leftrightarrow y$</p> $x = \frac{2y^2-1}{y^2+1}$ $\Rightarrow xy^2 + x = 2y^2 - 1$ $\Rightarrow x + 1 = 2y^2 - xy^2 = y^2(2-x)$ $\Rightarrow y^2 = \frac{x+1}{2-x}$ $\Rightarrow y = \sqrt{\frac{x+1}{2-x}} *$	M1 M1 M1 E1 [4]	(coULD start from g) Attempt to invert clearing fractions collecting terms in y^2 and factorising www

$\mathbf{2'(i)} \quad \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$ $= -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}} *$	M1 A1 M1 E1 [4]	Implicit differentiation (must show = 0) solving for dy/dx www. Must show, or explain, one more step.
$\mathbf{(ii)} \quad \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ $= -\left(\frac{8}{1}\right)^{\frac{1}{3}} \cdot 6$ $= -12$	M1 A1 A1cao [3]	any correct form of chain rule

<p>3 (i) Stretch in x-direction s.f. translation in y-direction 1 unit up</p>	M1 A1 M1 A1 [4]	(in either order) – allow ‘contraction’ dep ‘stretch’ allow ‘move’, ‘shift’, etc – direction can be inferred from $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ dep ‘translation’. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ alone is M1 A0
<p>(ii) $A = \int_{-\pi/4}^{\pi/4} (1 + \sin 2x) dx$</p> $= \left[x - \frac{1}{2} \cos 2x \right]_{-\pi/4}^{\pi/4}$ $= \pi/4 - \frac{1}{2} \cos \pi/2 + \pi/4 + \frac{1}{2} \cos (-\pi/2)$ $= \pi/2$	M1 B1 M1 A1 [4]	correct integral and limits. Condone dx missing; limits may be implied from subsequent working. substituting their limits (if zero lower limit used, must show evidence of substitution) or 1.57 or better – cao (www)
<p>(iii) $y = 1 + \sin 2x$ $\Rightarrow \frac{dy}{dx} = 2\cos 2x$ When $x = 0$, $\frac{dy}{dx} = 2$ So gradient at $(0, 1)$ on $f(x)$ is 2 \Rightarrow gradient at $(1, 0)$ on $f^{-1}(x) = \frac{1}{2}$</p>	M1 A1 A1ft B1ft [4]	differentiating – allow 1 error (but not $x + 2\cos 2x$) If 1, then must show evidence of using reciprocal, e.g. $1/1$
<p>(iv) Domain is $0 \leq x \leq 2$.</p>	B1 M1 A1 [3]	Allow 0 to 2, but not $0 < x < 2$ or y instead of x clear attempt to reflect in $y = x$ correct domain indicated (0 to 2), and reasonable shape
<p>(v) $y = 1 + \sin 2x \ x \leftrightarrow y$ $x = 1 + \sin 2y$ $\Rightarrow \sin 2y = x - 1$ $\Rightarrow 2y = \arcsin(x - 1)$ $\Rightarrow y = \frac{1}{2} \arcsin(x - 1)$</p>	M1 A1 [2]	or $\sin 2x = y - 1$ cao